MULTI-CRITERIA DECISION TOOL USING FIS MAMDANI vs FIS TAKAGI-SUGENO-KANG

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Abstract. *Competitive analysis of the characteristics of an organization using fuzzy logic represents a new approach because it is based on rules that are similar to human thinking. Due to the simplicity of the calculation of datasets uncertain fuzzy logic analysis indicated two ways defuzzification Mamdani FIS and FIS TSK to have a more realistic overview leading to more accurate forecasting. The paper presents the classic calculation and realization of graphs comparing the graphics mode performed with the software Matlab R2011b. The results are based on a comparative analysis between Mamdani FIS and FIS TSK. The purpose of this paper is to show how the practical realization of such an analysis type Multi-Input Single Output - MISO by showing the steps to take and interpretation of results. Type multi-criteria decisions of special interest because it covers a spread spectrum analysis data sets entry through flexibility, which gives robustness in various applications regardless of their field.*

Keywords: *fuzzy logic, FIS Mamdani, FIS Sugeno, competitiveness*.

JEL Classification: A12, C53, C63, D89

1. Introduction

Competitive analysis of the characteristics of an organization using fuzzy logic represents a new approach because it is based on rules that are similar to human thinking.
Because the simplicity of the calculation of datasets uncertain fuzzy logic indicated an analysis by two methods of defuzzification FIS Mamdani and FIS TSK (Takagi-Sugeno-Kang) to have an overview of more realistic leading to forecasting more accurate.
The paper presents the classic calculation and realization of graphics which are compared with the graphical software developed with Matlab R2011b. The results are based on a comparative analysis between Mamdani FIS and FIS TSK.
The purpose of this paper is to show how the practical realization of such analyzes multi input single output (Multi-Input Single Output - MISO) by highlighting the steps to take and interpretation of results.
Multi-criteria decisions of particular interest because it covers a spread spectrum analysis data sets entry through flexibility, which gives robustness in various applications regardless of their field.

2. The concept of Fuzzy Logic Systems

In 1965 Lotfi Zadeh Askar (professor at the University of California, Berkeley) introduced the term fuzzy logic to explain the theory of fuzzy sets [1].
Fuzzy logic continues working with a range of values in the range [0, 1]. Fuzzy set theory translates into quality human reasoning through simultaneous quantitative numerical expressions of numerical data and lexical knowledge [Cetean Răzvan 2014].

Mamdani type defuzzification methods [Yang Wang, Yanyan Chen, 2014; Florin Leon]**:**

1. *centroid of area COA (center of gravity)*

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This method returns an output defuzzification by calculating the center of symmetry of the area delimited by aggregating the consequences of such fuzzy set:

$$y_{COA} = \frac{\int_v y \cdot \mu_B(y) \cdot dy}{\int_v \mu_B(y) \cdot dy} \tag{1}$$

2. bisector of area BOA

The vertical line corresponds to the output generated by BOA dividing the aggregate fuzzy sets into two sub-regions of equal area. This can be expressed as:

$$\int_{\alpha}^{y_{BOA}} \mu_B(y) \cdot dy = \int_{y_{BOA}}^{\beta} \mu_B(y) \cdot dy \tag{2}$$

where $\alpha = \min\{v|v \in V\}, \beta = \max\{v|v \in V\}$. Sometimes resulting value by this method is identical to that generated by COA.

COA and BOA used in control applications because it does not produce jumps in the control surface.

3. smallest of maximum SOM

This method generates a clear exit by taking the lowest values that provide the maximum degree of belonging aggregate fuzzy set.

$$y_{SOM} = \min\{y|\mu_B(y) = \max(\mu_B(y))\} \tag{3}$$

4. largest of maximum LOM

This method generates a clear exit by taking the highest value to provide the maximum degree of belonging aggregate fuzzy set.

$$y_{LOM} = \max\{y|\mu_B(y) = \max(\mu_B(y))\} \tag{4}$$

5. mean of maximum MOM

In this defuzzification, the average maximum is taken as a clear exit.

$$y_{MOM} = \frac{y_{SOM} + y_{LOM}}{2} \tag{5}$$

MOM, LOM, SOM decision is used in applications as this may cause jumps in the control surface. The situation appears most advantageous method of execution when the item presents a finite number of positions that may be associated with strong linguistic terms of "singleton" of the output quantity [3].

Methods of defuzzification type Takagi-Sugeno:

a) weighted average WA

This method of defuzzification generates an output end result FIS Sugeno method of weighting symmetry centers of individual surfaces.

$$y_{WA} = \frac{\sum_{i=1}^{M} w_i \cdot y_i}{\sum_{i=1}^{M} w_i} \tag{6}$$

b) weighted sum WS

To reduce calculation WA, WS method only needs the right amount weighted outputs.

$$y_{WS} = \sum_{i=1}^{M} w_i \cdot y_i \tag{7}$$

In the study as a method of defuzzification for FIS Mamdani – centroid of area (COA), and for FIS TSK – weighted sum (WS).

3. Research methodology

To study use data from the branch of a multinational company from Romania in the trade of heating.

Context: An import company sells heating equipment using the distribution network of 31 outlets in the country. The marketing season of heating is from September 1 to November 30 (91 days or 14 weeks) and the period under review is 2013-2015. Demand is highest is 12 buc./zi, and the lowest is 1 buc./zi. It starts from an initial stock of 14 pieces. to be completed in two days to Metin continuity of supply for an amount equal to the difference between the initial stock and the stock at the end of the day. Stocks varies between 14 pc. and 6 at the end of the day. The maximum amount that can be supplied is 18 pcs. Efficiency and the minimum is 4 pcs.

If a request for 10 pieces. which is the supply capacity for an additional 8 pcs.

Table 1 Sales of heating equipment during the season 2013-2015

		2013						2014						2015					
		L	M	M	J	V	S	L	M	M	J	V	S	L	M	M	J	V	S
SEP	S1	2	3	5	1	1	1	10	3	0	4	2	0		1	5	0	6	1
	S2	6	7	7	1	4	0	6	2	7	1	2	0	10	8	9	1	4	2
	S3	6	1	0	3	10	4	4	5	5	0	4	2	1	3	6	1	1	0
	S4	5	3	10	4	7	0	11	4	0	1	3	2	6	1	10	2	5	4
	S5	12						10	9					5	6	8			
OCT	S1		8	7	0	7	1			2	4	1	4				1	1	1
	S2	8	7	5	1	1	1	1	2	3	2	0	0	8	0	5	0	4	0
	S3	9	7	7	3	2	4	6	8	6	1	0	1	5	2	3	3	6	1
	S4	12	2	12	8	2	0	10	6	0	2	2	2	10	2	4	10	2	5
	S5	6	2	1	8			12	5	8	3	0		8	12	3	7	4	
NOV	S1					11	0						1	1	1	1	2	3	2
	S2	5	3	7	3	1	0	4	5	5	1	7	0	12	0	4	2	2	0
	S3	3	6	4	1	0	1	1	0	2	1	1	1	1	4	2	0	3	2
	S4	1	1	1	9	1	0	7	1	6	7	1	0	1	1	9	2	7	0
	S5	5	1	3	5	2		2	2	3	1	2	3	0					

Methods (Mamdani and TSK) involve the following steps:

Phase 1. Define **input variables ($v_{intrare}$)** and **output variables ($v_{ieșire}$)**;

Phase 2. Establishing **basic ranges**;

Phase 3. Determining the **type of fuzzy numbers** (triangular, trapezoidal, Gaussian, bell, sigmoid, left ramp, the right ramp).

Phase 4. Establishing **linguistic variable** size (LV) for the input variables.

Phase 5. Building in **Matlab R2011b** depending on membership type chosen variables.

Phase 6. Establishing **ruleset.**

Phase 8. Running the application and **interpretation of data**.

3.1. Analysis using fuzzy logic Mamdani FIS in Matlab

This method uses the **min** operator implication and compose operator **min-max** [Dosoftei Constantin-Cătălin, 2009].

The three variables that make up the issue request (*x*), stocks (*y*) and supply (*z*) are further treated separately [6].

Supply branch operates under the following **rules**:

R1: IF demand is LOW and supply is LOW and the stock is LOW;

R2: IF demand is LOW and supply is HIGH and the stock is LOW;

R3: IF demand is HIGH and supply is LOW and the stock is HIGH;

R4: IF demand is HIGH and supply is HIGH and the stock is HIGH;

REQUEST is composed of two fuzzy sets: 1 - LOW and 2 - HIGH. It is known that the greatest demand is 12 and the lowest pcs./day 1 pcs./day. By conducting obtain replacements and solving for *x* values.

$$\mu_{request-low(x)}\begin{cases} 1 & x \le 1 \\ \dfrac{12-x}{11} = \dfrac{12-10}{11} = 0{,}182 & 1 \le x \le 12 \\ 0 & x \ge 12 \end{cases} \tag{8}$$

$$\mu_{request-high(x)}\begin{cases} 0 & x \le 1 \\ \dfrac{x-1}{11} = \dfrac{10-1}{11} = 0{,}818 & 1 \le x \le 12 \\ 1 & x \ge 12 \end{cases} \tag{9}$$

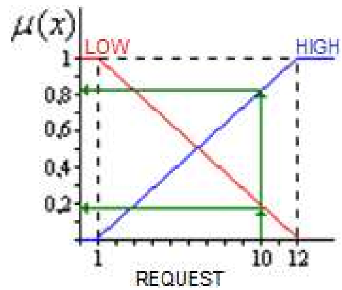


Figure 1 Chart demand (x) by the classical method

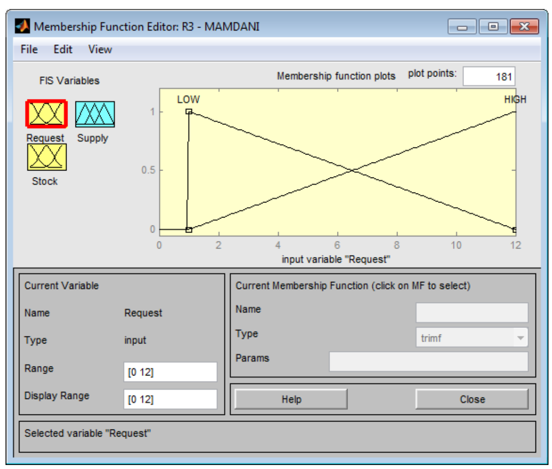


Figure 2 Mamdani FIS demand diagram with triangular membership function - Matlab R2011b, fuzzy

STOCKS are composed of two fuzzy sets: 1 - LOW and 2 - HIGH. It is known that stocks range from 14 pcs. and 6 at the end of the day. By conducting obtain replacements and resolution values for *y*.

$$\mu_{stocks-low(y)}\begin{cases} 1 & y \le 6 \\ \dfrac{14-y}{8} = \dfrac{14-8}{8} = 0{,}75 & 6 \le y \le 14 \\ 0 & y \ge 14 \end{cases} \tag{10}$$

$$\mu_{stocks-high(y)}\begin{cases} 0 & y \le 6 \\ \dfrac{y-6}{8} = \dfrac{8-6}{8} = 0{,}25 & 6 \le y \le 14 \\ 0 & y \ge 14 \end{cases} \quad (11)$$

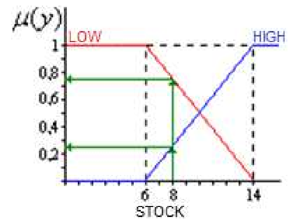


Figure 3 Chart risk (y) by the classical method

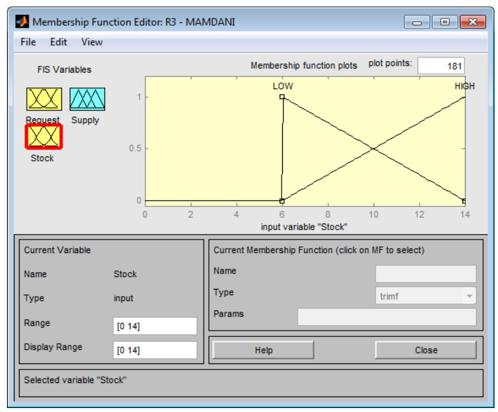


Figure 4 Mamdani FIS risk diagram with triangular membership function - Matlab R2011b, fuzzy

SUPPLY is composed of two fuzzy sets: 1 - LOW and 2 - HIGH. *z* so we must determine if a request for 10 pcs. which is the supply capacity for an additional 8 pcs.

Determining the value of *z* by using the *min*.

$$\mu_{\sup ply-low(z)}\begin{cases} 1 & z \le 4 \\ \dfrac{18-z}{14} & 4 \le z \le 18 \\ 0 & z \ge 18 \end{cases} \quad (12)$$

$$\mu_{\sup ply-high(z)}\begin{cases} 0 & z \le 4 \\ \dfrac{z-4}{14} & 4 \le z \le 18 \\ 1 & z \ge 18 \end{cases} \quad (13)$$

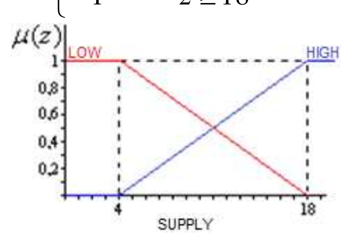


Figure 5 Chart supply (*z*) by the classical method

Determine coefficients a_1, a_2 for z:

$$\frac{a_1 - 4}{14} = 0{,}182 \Rightarrow a_1 = 6{,}55$$
$$\frac{a_2 - 4}{14} = 0{,}75 \Rightarrow a_2 = 14{,}5 \qquad (14)$$

So we get to a member function following components:

$$\mu_{(z)} \begin{cases} 0{,}182 & z \le 6{,}55 \\ \dfrac{z-4}{20} & 6{,}55 \le z \le 14{,}5 \\ 0{,}75 & z \ge 14{,}5 \end{cases} \qquad (15)$$

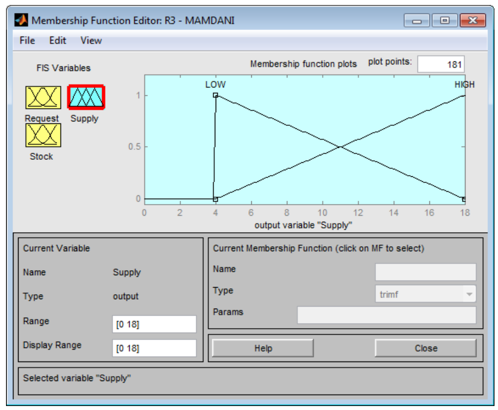


Figure 6 Mamdani FIS supply diagram with triangular membership function - Matlab R2011b, fuzzy

Defuzzification

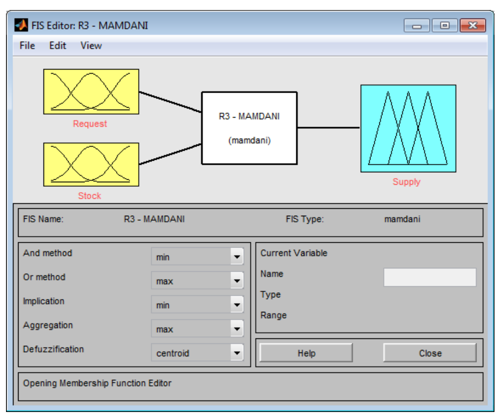


Figure 7 Mamdani FIS features used - Matlab R2011b, fuzzy

The features you use this analysis to FIS are Mamdani:

- connector: AND;
- operator: ***min*** for the implication;
- operator: ***min-max*** for the compose;
- defuzzification: centroid of area;

Rule base contains the type ruleset *"if ...then ..."* expert established and defined the fuzzy input and output variables. Based fuzzy logic rules they match the description language.

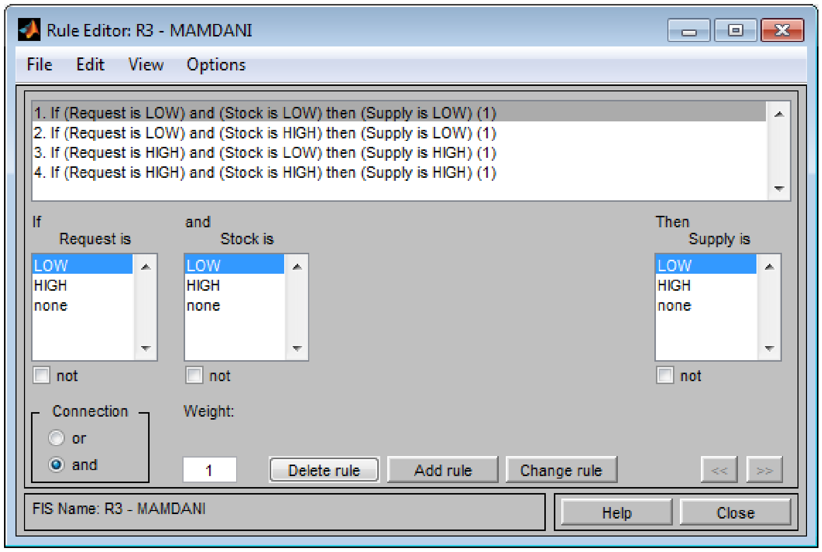


Figure 8 Supply setting rules Mamdani FIS - Matlab R2011b, fuzzy

Use method for defuzzification COA (centroid of area).

$$M_1 = \int_0^{6,55} 0,182 \cdot z \cdot dz = 0,182 \cdot \frac{z^2}{2}\Big|_0^{6,55} = 3,895 \qquad (16)$$

$$M_2 = \int_{6,55}^{14,5} \frac{z-4}{20} \cdot z \cdot dz = 0,05 \cdot \frac{z^3}{3}\Big|_{6,55}^{14,5} - 0,2 \cdot \frac{z^2}{2}\Big|_{6,55}^{14,5} = 48,993 \qquad (17)$$

$$M_3 = \int_{14,5}^{18} 0,182 \cdot z \cdot dz = 0,182 \cdot \frac{z^2}{2}\Big|_{14,5}^{18} = 42,656 \qquad (18)$$

Next calculate the area of each second area.

$$A_1 = 6,55 \cdot 0,182 = 1,19 \qquad (19)$$

$$A_2 = \frac{(0,182 + 0,75) \cdot (14,5 - 6,55)}{2} = 3,706 \qquad (20)$$

$$A_3 = (18 - 14,5) \cdot 0,75 = 2,625 \qquad (21)$$

Finally we get for the *z* value:

$$z = \frac{3,895 + 42,993 + 42,656}{1,19 + 3,706 + 2,625} = 12,703 \qquad (22)$$

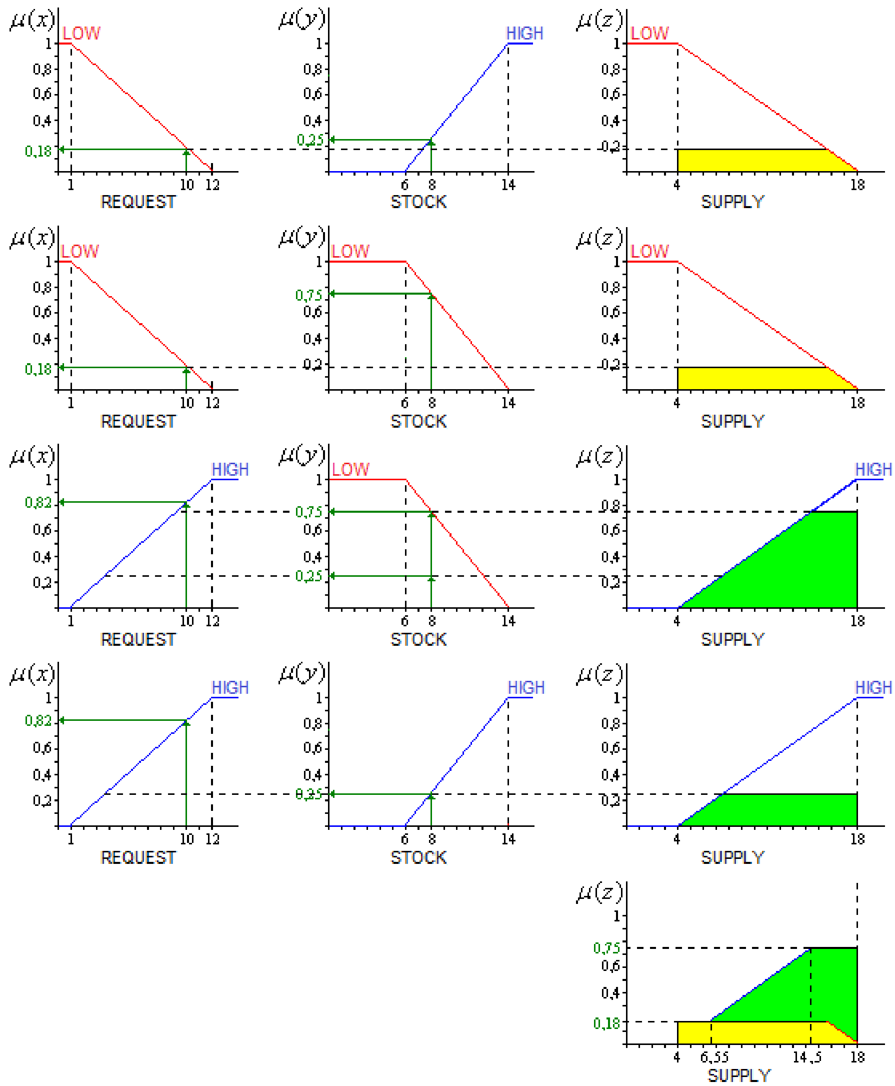


Figure 9 Mamdani FIS classic graphical method of defuzzification using the centroid of area

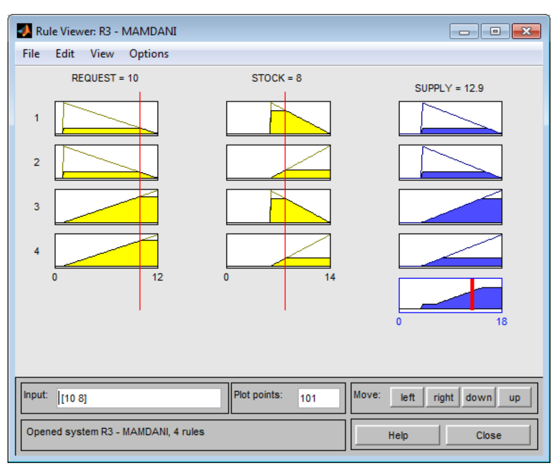


Figure 10 Mamdani FIS defuzzification using centroid of area - Matlab R2011b, fuzzy

Comparing the calculated result $z = 12{,}703$ equation (1.19) calculated using Matlab R2011b at where $z = 12{,}9$ observed that the error $e = -1{,}53$ it is acceptable which shows that this method is robust, saving time in making management decisions on integrated systems in concept.

With software - Matlab R2011b can be viewed in 2D the wave shapes of input variables respectively *request* and *stock*.

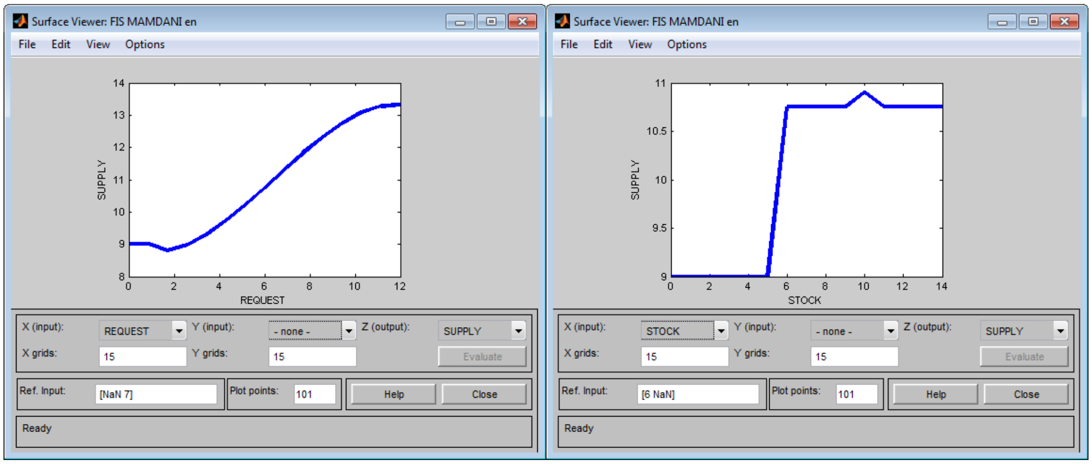
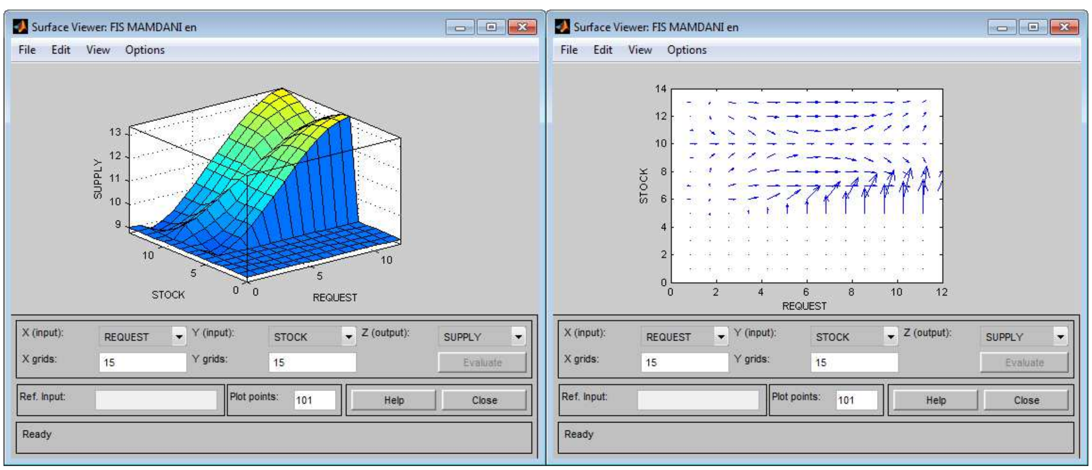


Figure 11 Mamdani FIS waveform to the input variables (request and stock) - Matlab R2011b, fuzzy

You can also switch from 2D view to 3D view where the two input variables are correlated with variable output respectively supply, which leads to a superior interpretation of results.



(a) (b)

Figure 12 **Shape of the surface (a) and quiver (b) Mamdani FIS for the output variables - Matlab R2011b, fuzzy**

The dynamics of variation present purpose of Action input variables 3D surface. Figure 12 (b) shows trends in the areas quiver as possible as the target (light-colored areas Figure 12 (a)), so positive targets. There are situations when the same surface Figure 12 (a) quiver to have the sense of action opposite to that shown, so travel from favorable areas to areas devaforabile dark areas in Figure 12 (a)).

3.2. Analysis of fuzzy logic Takagi-Sugeno FIS using the Matlab-Kang

Takagi-Sugeno fuzzy model-Kang Mamdani model is similar except that the output is characterized by equations [8]:

a) Takagi-Sugeno-Kang fuzzy model zero order (constant equation):

$$IF\ (x_1 \text{ is } A_1)^*\ (x_2 \text{ is } A_2)^*...^*\ (x_n \text{ is } A_n)\ THEN\ z = k \qquad (23)$$

b) Takagi-Sugeno-Kang fuzzy model of the order of one (linear equation):

$$IF\ (x_1 \text{ is } A_1)^*\ (x_2 \text{ is } A_2)^*...^*\ (x_n \text{ is } A_n)\ THEN\ z = p_1{}^*z_1+...+ p_n{}^*z_n+ q \qquad (24)$$

The input variables shown in Figures 1, 3 and 5 are retained numerical, linguistic variable and membership function. The function of the supply output diagram is shown in Figure 13.

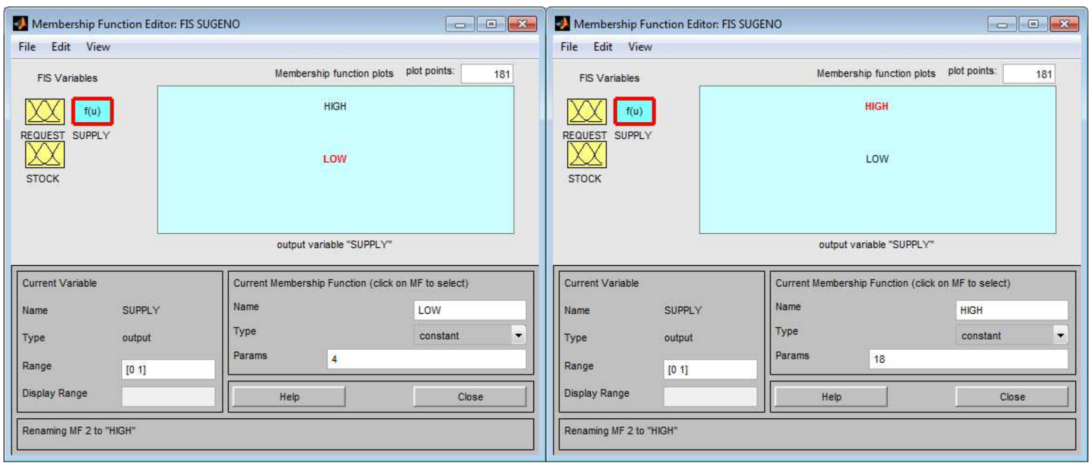


Figure 13 Chart constant supply function FISA TSK - Matlab R2011b, fuzzy

Supply branch operates under the following **rules**:

R1: IF request is LOW and stock is HIGH then supply = request – stoks

$$\alpha_1 = \mu_{requst-low} \cap \mu_{stock-high}$$
$$\alpha_1 = \min(\mu_{requst-low}[10] \cap \mu_{stock-high}[8]) \qquad (25)$$
$$\alpha_1 = \min(0{,}1818;0{,}25) \Rightarrow \alpha_1 = 0{,}1818$$

$$z_1 = 10 \qquad (26)$$

R2: IF request is LOW and stock is LOW then supply = request

$$\alpha_2 = \mu_{requst-low} \cap \mu_{stock-low}$$
$$\alpha_2 = \min(\mu_{requst-low}[10] \cap \mu_{stock-low}[8]) \qquad (27)$$
$$\alpha_2 = \min(0{,}1818;0{,}75) \Rightarrow \alpha_2 = 0{,}1818$$

$$z_2 = 10 \qquad (28)$$

R3: IF request is HIGH and stock is HIGH and supply = request

$$\alpha_3 = \mu_{requst-high} \cap \mu_{stock-high}$$
$$\alpha_3 = \min(\mu_{requst-high}[10] \cap \mu_{stock-high}[8]) \qquad (29)$$
$$\alpha_3 = \min(0{,}818;0{,}25) \Rightarrow \alpha_3 = 0{,}25$$

$$z_3 = 10 \qquad (30)$$

R4: IF request is HIGH and stock is LOW and supply = 1,25 * request

$$\alpha_4 = \mu_{\text{request}-high} \cap \mu_{stock-low}$$

$$\alpha_4 = \min(\mu_{\text{request}-high}[10] \cap \mu_{stock-low}[8]) \qquad (31)$$

$$\alpha_4 = \min(0{,}818;0{,}75) \Rightarrow \alpha_4 = 0{,}75$$

$$z_4 = 1{,}25 \cdot 10 = 12{,}5 \qquad (32)$$

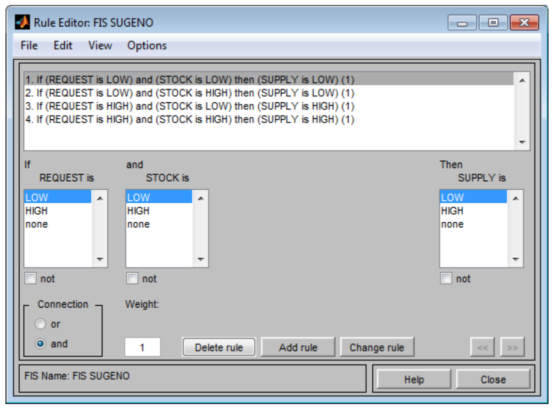


Figure 14 FIS TSK rules establishing the supply - Matlab R2011b, fuzzy

Method *weighted sum - WS* usually only needs the weighted sum of outputs:

$$y_{WS} = \sum_{i=1}^{M} \alpha_i \cdot z_i = \alpha_1 \cdot z_1 + \alpha_2 \cdot z_2 + \alpha_3 \cdot z_3 + \alpha_4 \cdot z_4 \qquad (33)$$

$$y_{WS} = 0{,}1818 \cdot 10 + 0{,}1818 \cdot 10 + 0{,}25 \cdot 10 + 0{,}75 \cdot 12{,}5 = 15{,}51 \qquad (34)$$

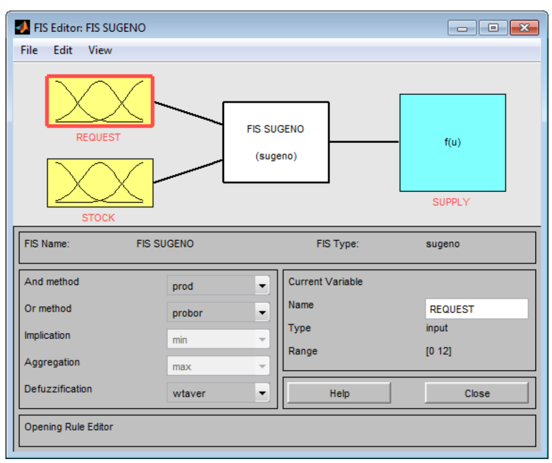


Figure 15 FIS TSK features used weighted sum - Matlab R2011b, fuzzy

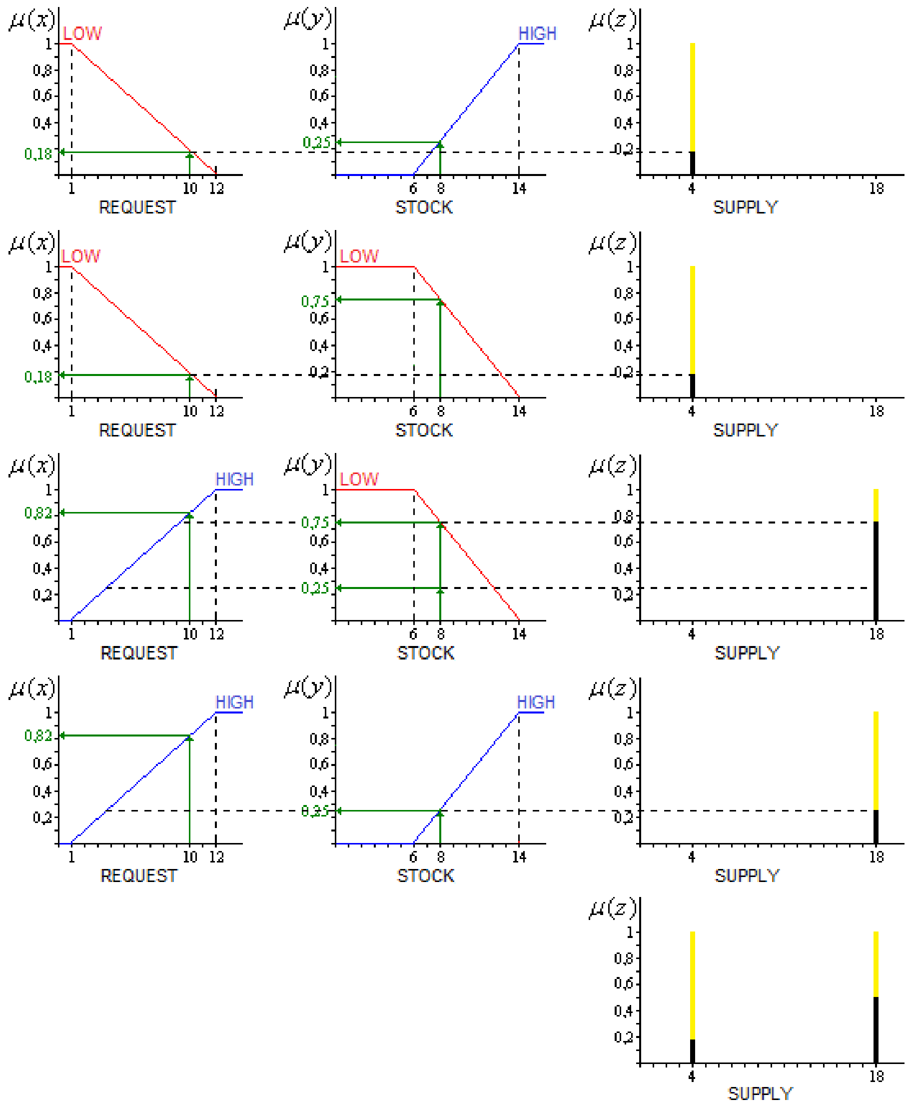


Figure 16 FIS TSK classic graphical method using the weighted sum defuzzification

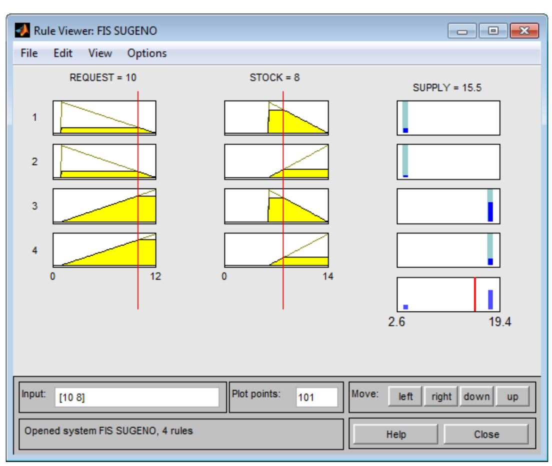


Figure 17 FIS TSK using the weighted sum defuzzification – Matlab R2011b, fuzzy

Comparing the calculated result $z = 15{,}51$ equation (1.31) calculated using Matlab R2011b at where $z = 15{,}5$ observed that the error $e = 0{,}07 < 1$ is very small, unity, and make this method to be very robust systems management decisions on the integrated design extremely precise.

2D the wave shapes for the input variables respectively request and stock are shown in Figure 18.

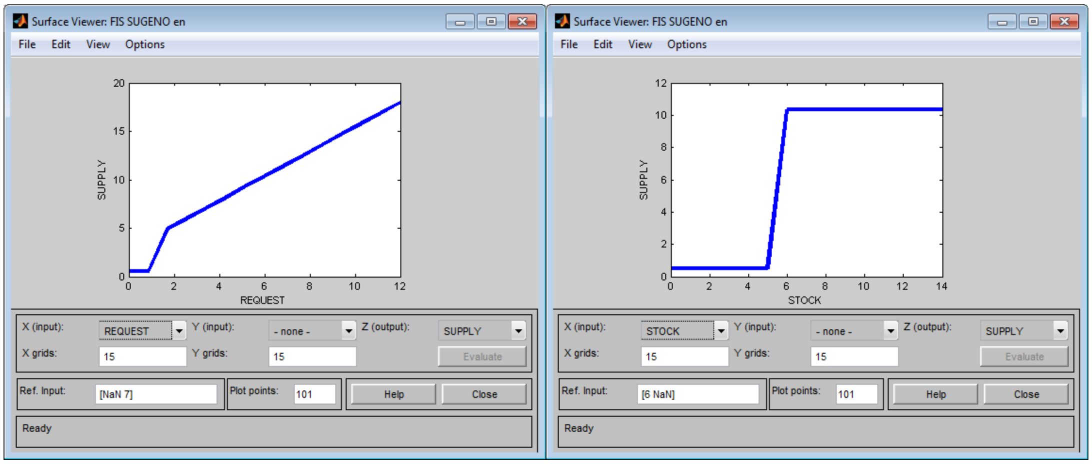


Figure 18 FIS TSK waveform to the input variables (request and stock) – Matlab R2011b, fuzzy

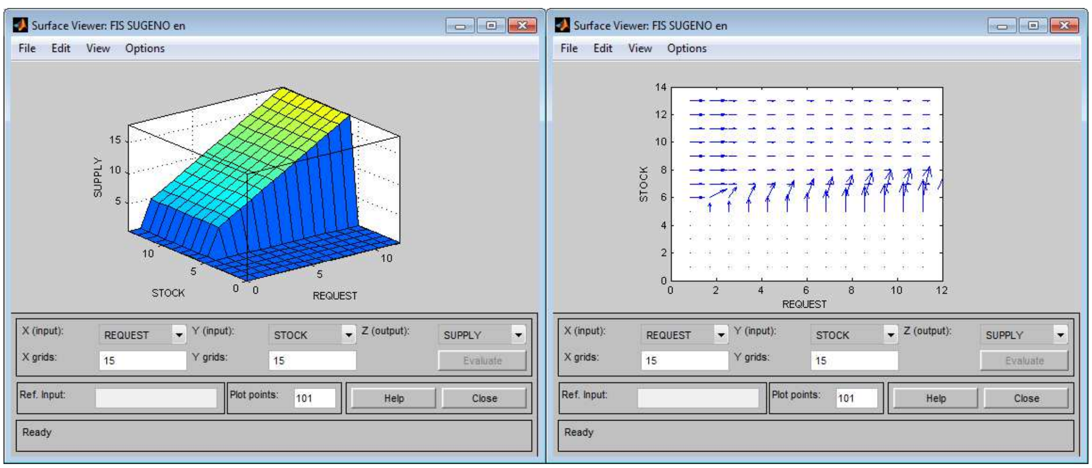


Figure 19 Shape of the surface and quiver TSK display for the output variables – Matlab R2011b, fuzzy

Conclusion

The waveform of the input variables supply demand in the two cases differ in the areas of start stock-supply and if differ only in the end zone. Differentiation occurs when two surfaces 3D models by smoothing the surface slope (FIS TSK) from the surface showing Mamdani sharing this surface slope. Differentiation slope areas leave room for interpretation and decision maneuver on target. It is possible that the best decision to be taken by unifying results areas (consider the average of the two models in those areas). TSK area is characterized by sudden leaps to the Mamdani showing a smooth transition.

Table 2 Mamdani FIS models and TSK errors for the the sample analyzed

	Calculat	Matlab	Eroare
FIS Mamdani	12,703	12,9	-1,53%
FIS TSK	15,511	15,5	0,07%

In terms of the outcome of the TSK defuzzification we can say that FIS is more accurate in forecasting the output error is much smaller. Both models fit in a field error $e = (0 \div \pm 1{,}5)$.

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